ABSORBENCY OF PAPER TOWELS

9. General Factorial Model

In this section General Factorial Analysis will be used to investigate the effects of all possible combinations of towel brand and immersion time on absorbency of the three brands. In particular, the main effects for brand and time are shown to be highly significant, while the interaction is shown to be not significant.

9.1 Absorbency of Paper Towels Experiment as a Factorial Experiment

The absorbency of paper towels experiment is an example of a factorial experiment. A factorial experiment consists of several factors (brand type, immersion time) which are set at different levels, and a response variable (weight of water absorbed). The purpose of the experiment is to assess the impact of different combinations of the levels of brand and immersion time on the weight of water absorbed by the paper towel. Analysis of variance allows us to test the null hypothesis that brand type and time have no impact on absorption. As the experiment involved replications, so that responses are available from more than one subject at each combination of levels of brand type and time, the presence of interaction can be assessed.

The General Factorial Procedure available in SPSS 8.0 provides regression analysis and analysis of variance for one dependent variable by one or more factors or variables. The SPSS data file used for this study is available in the SPSS file *towel.sav* located on the FTP server in the Stat337 directory. In the data file, variables include brand, time and weight of water absorbed. The two-predictor variables in this study, brand type and time level, are categorical, which means they should be entered as factors in the GLM General Factorial procedure.

To produce the output for your data, select *SPSS Instructions* in the problem menu now. Here, we will display and analyze the output for our data.

9.2 The ANOVA Output for the Absorbency of Paper Towels Experiment

Analysis of variance allows us to test the null hypothesis that brand type and time have no impact on absorbency. There are four sources of variation in the experiment: the main effects of *Brand* and *Time*, the interaction effect, and the error variation. Corresponding to these four sources, there are three null hypotheses that may be tested:

1. H_0: No main effect of *Brand*
2. H_0: No main effect of *Time*
3. H_0: No interaction effect between *Brand* and *Time*

The GLM General Factorial procedure in SPSS produces the following output for the experiment:

Tests of Between-Subjects Effects

Dependent Variable: WATER

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	81.303[a]	8	10.163	58.632	.000
Intercept	6303.617	1	6303.617	36367.021	.000
BRAND	74.712	2	37.356	215.517	.000
TIME	4.915	2	2.458	14.178	.000
BRAND * TIME	1.676	4	.419	2.417	.067
Error	6.240	36	.173		
Total	6391.160	45			
Corrected Total	87.543	44			

a. R Squared = .929 (Adjusted R Squared = .913)

The table contains rows for the components of the model that contribute to the variation in the dependent variable. The row labeled *Corrected Model* contains values that can be attributed to the regression model, aside from the intercept. The sources of variation are identified as *Brand*, *Time*, *Brand*Time* (interaction), and *Error*. *Error* displays the component attributable to the residuals, or the unexplained variation. *Total* shows the sum of squares of all values of the dependent variable. *Corrected Total* (sum of squared deviations from the mean) is the sum of the component due to the model and the component due to the error.

The total number of degrees of freedom is 44 =45-1, which is one less than the number of towel sheets tested. Two degrees of freedom are associated with *Brand*, which is one less than the number of levels of this factor, and similarly 2 degrees of freedom for *Time*. The interaction factor *Brand*Time* degrees of freedom equals 4 = (3-1)(3-1), the product of the degrees of freedom associated with the two factors. Interaction can be thought of as the joint effect of the two factors.

According to the output, the model sum of squares is 81.303 and the error sum of squares is 6.240. The total sum of squares (corrected total) is 87.543. Notice a very small contribution of error in the total sum of squares. The p-value of the F-test for the model is reported as 0.000 indicating convincing evidence of an effect of at least one of the factors on absorbency.

The sum of squares for the brand factor is estimated to be only 74.712, an extremely large value compared to the total sum of squares. The value of the F-statistic equal to 215.517 and p-value of the F-test reported as 0.000 indicate very strong evidence of effect of brand on absorbency. Indeed, in all graphical displays and numerical summaries we found strong evidence of brand effect on the absorbency.

The sum of squares due to time is also 4.915, a very small contribution in the total sum of squares of 87.543. The value of the F-statistic is 14.178 with the corresponding reported p-value of 0.000. *Time* main effects are also statistically significant, although they are not that strong as the main effects due to brand factor.

The p-value of the interaction term *Brand*Time* is equal to 0.067, indicating a weak evidence of an interaction between the two factors.

9.3 Exploring the Interaction Effects

To further explore the interaction effects, we examine the table of estimated marginal means and the profile plot of the same values displayed below.

Report

WATER

BRAND	TIME	Mean	N	Std. Deviation
1.00	3.00	9.3800	5	.6419
	5.00	10.4200	5	.6058
	10.00	10.7200	5	.1924
	Total	10.1733	15	.7658
2.00	3.00	11.5800	5	.4712
	5.00	12.2000	5	.4528
	10.00	12.2800	5	.1304
	Total	12.0200	15	.4814
3.00	3.00	13.1600	5	.4037
	5.00	13.4000	5	.3162
	10.00	13.3800	5	.1924
	Total	13.3133	15	.3137
Total	3.00	11.3733	15	1.6739
	5.00	12.0067	15	1.3408
	10.00	12.1267	15	1.1411
	Total	11.8356	45	1.4105

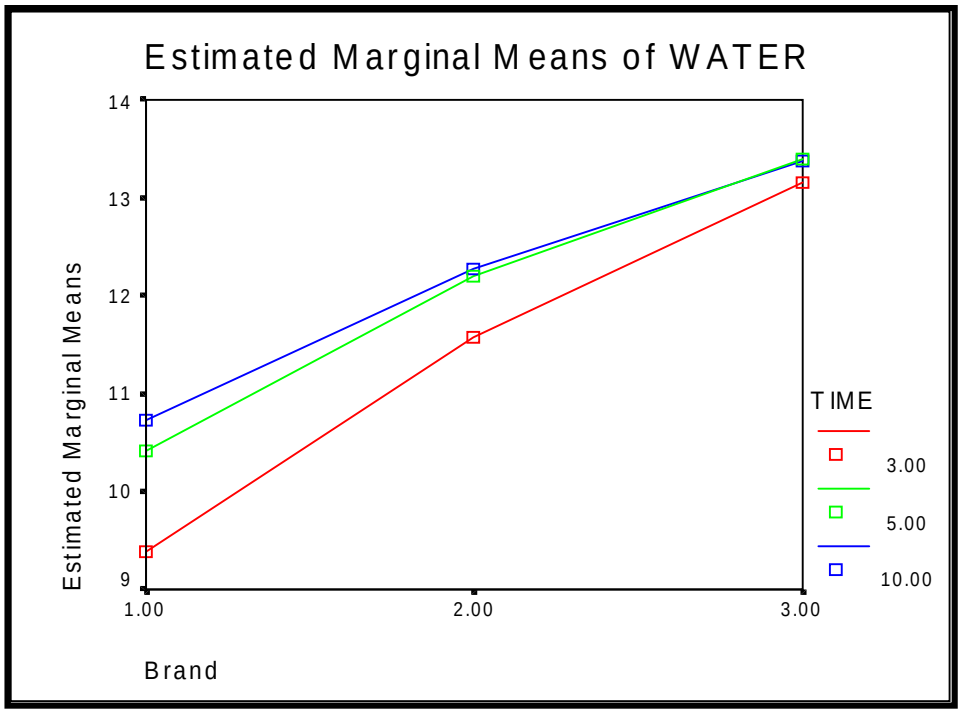
The table shows the means of amount of water absorbed for each combination of levels of the two factors. The mean amount of water absorbed increases when taken across time levels. There is almost no change in the mean when time increases from the 5 second level to the 10 second level.

The overall means among the three brands, 10.1733, 12.0200, and 13.3133 differ significantly, which indicates very strong brand main effects.

The impact of immersion time can be noticed by comparing the mean amount of water absorbed at time level of 3 and 5 seconds. The differences indicate that the time main effects seem to be statistically significant. On the other, the differences for the time levels of 5 and 10 seconds are very small for each brand.

As you can see, the standard deviation for each brand decreases when taken across the three time levels. Moreover, the table shows that the highest mean amount of water absorbed is achieved for the brand 3. The lowest mean amount of water absorbed is achieved for the brand 1.

Now we examine the interaction effects with a profile plot. In general, profile plots (interaction plots) are useful for comparing marginal means in your model. A profile plot is a line plot in which each point indicates the estimated marginal mean of a dependent variable at one level of a factor. The plot for our data is displayed below.



The plot indicates that the brand 3 has the best absorbency, and the brand 1 has the worst across the three time levels. The lines for the time levels 5 and 10 seconds are almost identical, there is no change in the mean amount of water absorbed as time changes from 5 seconds to 10 seconds. It indicates that water is absorbed very fast. The water is absorbed much slower for the brand 1.

The lines in the above graph indicate that there is a weak interaction between brand and time. The strongest interaction effect (biggest deviation from zero) is shown for the brand level 1 with time levels of 3 and 5 seconds. This corresponds to the point where the above graph displays the greatest degree of non-additivity.

9.4 Multiple Comparisons

The results in the ANOVA table in the previous section clearly indicate a difference among the means for the brand and the time factors, but they do not identify just which mean differs from another. The results of the Tukey method for each factor are presented below.

Multiple Comparisons

Dependent Variable: WATER

Tukey HSD

(I) BRAND	(J) BRAND	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-1.8467*	.152	.000	-2.2183	-1.4751
	3.00	-3.1400*	.152	.000	-3.5116	-2.7684
2.00	1.00	1.8467*	.152	.000	1.4751	2.2183
	3.00	-1.2933*	.152	.000	-1.6649	-.9217
3.00	1.00	3.1400*	.152	.000	2.7684	3.5116
	2.00	1.2933*	.152	.000	.9217	1.6649

Based on observed means.

*. The mean difference is significant at the .05 level.

As you can see, there are significant differences among the three brands. The p-values for all the comparisons between the brands are reported as zero.

The results of the Tukey test for the time factor are displayed below.

Multiple Comparisons

Dependent Variable: WATER

Tukey HSD

(I) TIME	(J) TIME	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
3.00	5.00	-.6333*	.152	.001	-1.0049	-.2617
	10.00	-.7533*	.152	.000	-1.1249	-.3817
5.00	3.00	.6333*	.152	.001	.2617	1.0049
	10.00	-.1200	.152	.712	-.4916	.2516
10.00	3.00	.7533*	.152	.000	.3817	1.1249
	5.00	.1200	.152	.712	-.2516	.4916

Based on observed means.

*. The mean difference is significant at the .05 level.

The above results confirm the conclusions we have reached before. The difference between the time level of 3 seconds and 5 seconds is significant with the p-value of .001. The same holds for the time level of 3 seconds and 10 seconds. However, the difference between the time levels of 5 and 10 seconds is found to be non-significant. Indeed, the graphical displays discussed in Section 7 indicated that the paper towels absorb water very fast. This is why there is a significant difference in the mean weight of water absorbed between time level of 3 seconds and time level of 5 seconds. As soon as some saturation has been achieved, there is very little change in the amount of water absorbed as time elapses.