ABSORBENCY OF PAPER TOWELS

15. Brief Version of the Case Study

15.1 Problem Formulation

In this case study you will be involved in an experiment of comparing the absorbency of paper towels of three brands. We have selected only three brands to minimize the amount of time required to collect the data, but you can carry out the experiment with a larger number of brands. The instructions provided in the study will allow you to compare the absorbency of the brands selected by you and examine your own data.

We will use the experiment to demonstrate the GLM General Factorial procedure available in SPSS. You will have to design the experiment, collect the data, enter the data into SPSS, carry out the statistical analysis, and formulate your conclusions. If you do not wish to be involved in the data collection process, you can use our data. The data will be analyzed using SPSS version 8.0.

15.2 Selection of Factors

Absorbency is one of the most important characteristics of good quality paper towels. It shows how much fluid a particular brand of paper towel holds. In order to obtain reliable conclusions, we have to design the experiment very carefully.

In the experiment the sheets of paper will be immersed in water for a specified time period and the weight of water absorbed by the paper towel will be calculated as the difference between the weight of wet sheet and the dry sheet. We refer to the time a sheet tested has been immersed in water as *immersion time*.

In this setting, the absorbency is affected by several factors such as the material characteristics, contact time, sheet size, liquid type, liquid temperature, surrounding temperature and humidity, and many others.

Only some of the factors can be controlled. For example, we cannot control surrounding humidity and temperature during the experiment. We will neutralize the uncontrollable factors in the experiment by using randomization.

It seems that brand type and contact time are the most important factors affecting absorption. Therefore, we will use them as factors in our experiment. The response variable is the weight of water absorbed by a towel.
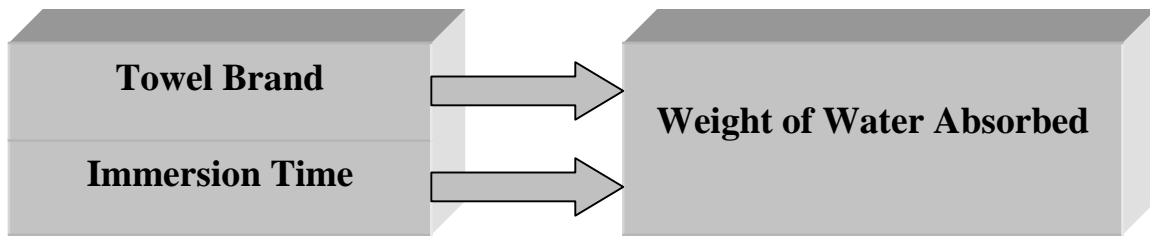
The towel brand factor will occur at three levels 1, 2, 3, and contact time will occur at three levels: 3 seconds, 5 seconds, and 10 seconds.

Write the names of these three varieties in the table below.

Factor	Level	Description
Brand	1	
	2	
	3	
Time	1	
	2	
	3	

15.3   Obtaining Random Samples of Paper Towels

Can we consider a roll of paper towel purchased at a store to be representative of all rolls manufactured by a particular manufacturer? Of course, not. In order to obtain representative rolls, we would have to obtain the rolls directly in the factory with a random mechanism. The best we can do here is to obtain a roll of each brand and then test a random sample of several sheets from each roll. It goes without speaking that the tested sheets on a particular roll do not comprise a random sample from the population of all sheets of that brand. However, if we assume that the rolls purchased at a store are representative of each brand, and a random mechanism is used to select sheets from each roll, our inferences should be valid for all rolls of each brand.

We will select a simple random sample of sheets from each roll according to the following procedure:

1.   Carefully tear off all the sheets on the roll. Set aside any sheets that are severely damaged in the process. Don't get in a hurry - do it right. All the undamaged sheets constitute the population. Assume that N is the number of undamaged sheets in your population.

2.   With a soft-tip marking pen, number the undamaged sheets with a small number in the upper right corner of each sheet.

3.   Use the table of random numbers to select randomly 15 non-repeated numbers from the set of N numbers. The selected numbers indicate the sheets from the brand population for absorbency testing.

Repeat the above steps for each brand of paper towels. The simple random samples of sheets are ready for testing.

15.4   How will the Absorbency be measured?

Before we will carry out the experiment, it is necessary to specify how the absorbency will be measured. In this section we will demonstrate how the basic measurement in our experiment will be taken.

If the experiment is carried out in a team, one team member needs to keep track of time, while the other does the following:

1.   Select a paper towel and fold it twice, to one-fourth its original size.

2.   Put the dry paper towel in the dry bowl, put the bowl on the balance, and record the total weight. The combined weight of the bowl and the dry sheet will be called the **dry weight**.

3.   Holding the sheet in the tongs, immerse the sheet completely in the bucket of water for 3, 5, or 10 seconds depending on which time level is used.

4.   With the tongs, hold the wet sheet above the water, letting it drip for 20 seconds. While doing this, remove the bowl from the balance and reset the balance.

5.   Place the wet sheet back in the bowl, place the bowl and sheet again on the balance, and record the total weight. The combined weight of the bowl and the wet towel will be called the **wet weight**.

6.   Discard the wet sheet and dry the bowl with an extra paper towel.

The weight of the water absorbed by the paper towel is the wet weight minus the dry weight. Before you start measuring the absorbency for the selected sheets, you should try obtaining this measurement on a sheet or two for practice.

15.5   Randomization

In the first part of the experiment we have selected three rolls of paper towels, one from each brand. We have assumed the rolls to be representative of their corresponding populations. Then we selected randomly 15 sheets from each roll. Now in order to examine the effects of brand type and contact time on the amount of water absorbed, it is necessary to assign randomly the 15 sheets from each brand to the three treatment groups corresponding to the three time levels.

As this experiment has two factors: brand (with three levels) and the contact time (with three levels), this creates 3 x 3 = 9 factor-level combinations, as represented by the cells in the following table.

Levels	General Description	Detailed Description
(1,1)		brand 1 and time level 1
(1,2)		brand 1 and time level 2
(1,3)		brand 1 and time level 3
(2,1)		brand 2 and time level 1
(2,2)		brand 2 and time level 2
(2,3)		brand 2 and time level 3
(3,1)		brand 3 and time level 1
(3,2)		brand 3 and time level 2
(3,3)		brand 3 and time level 3

This is an example of a two-factor experiment with replication.

To minimize the effect of uncontrollable factors, it is very important that the 15 sheets from each brand (experimental units) are assigned at random to the three treatment groups (three time levels). This technique gives each of the 15 sheets an equal chance to be chosen to each of the 3 time levels.

In the first step of randomization, it is necessary to assign labels to the experimental units. Two digits are needed to label each of the 15 sheets, so we use labels 01, 02, 03, ..., 15 for each of the three brands. As we assign 15 sheets from a particular brand to the three time levels, exactly five sheets will be assigned randomly to each treatment combination.
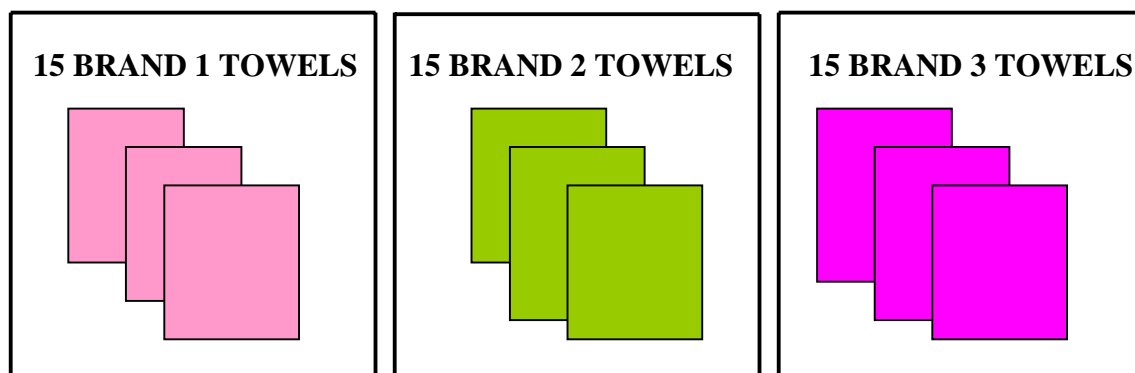
Now obtain a long sequence of random numbers between 1 and 15. The sequence can be obtained either from the table of random numbers (numbers different from the integers between 1 and 15 are disregarded) or by random number generation feature in the statistical software (integer uniform distribution with possible values between 1 and 15).

The first number in the sequence between 1 and 3 indicates the brand we will be selecting a towel sheet from. The next number between 1 and 15 indicates the number of sheet from the brand. Continue using the numbers until all the 15 sheets of each brand have been assigned one of the three time levels. Remember that each time level is to be used exactly five times for each of the three brands. That is, after five 1s have appeared for a particular brand, skip over the remaining 1s. Use the figure below to record the treatment-combination assignments as they are determined.

Test Number	Brand Number	Sheet Number	Time Level
1	2	6	2
2	3	5	3
3	1	11	1
4	1	4	2
5			1
...			
44			
45			

When you are finished, for each brand you should have 15 towel sheets labeled as five 1s, five 2s, and five 3s. The random assignment of the experimental units (towel sheets) to the treatment combinations is now complete.

Now work in teams to carefully make the absorbency measurements described in Section 4. Use the following table to record the results of your experiment.

TEST	BRAND	TIME	DRY WEIGHT	WET WEIGHT
1	1			
2	1			
	1			
	1			
15	1			
16	2			
17	2			
	2			
	2			
30	2			
31	3			
32	3			
45	3			

In the *Dry Weight* column of the above table, record the measured weight of the dry sheet and bowl together. In the *Wet Weight* column, record the measured weight of the wet sheet and bowl together.

Launch SPSS. In the new worksheet, name the first four columns *Test, Brand, Drywt*, and *Wetwt*. Remember to use numbers (1, 2, and 3), not letters (A, B, and C) for the variable *Brand*. When you have finished typing the data and have double-checked it, save the data set onto your diskette.

We will now create a new variable, *Water*, to reflect the weight of the water absorbed by each paper towel. *Water* for each row (sheet) is the difference *Wetwt-Drywt*, and can be created as follows:

1.   Click on the *Transform* menu, and then select *Compute...* from the submenu. A *Compute Variable* dialog box will appear.

2.   In the *Target Variable* entry box, enter the name of the variable, *Water*.

3.   In the *Numeric Expression* entry box type "*Wetwt-Drywt*".

4.   Click OK.

In a moment, the new variable, *Water*, should appear in your worksheet. Now we are ready to carry the statistical analysis of the data.

The data from our experiment are available in the SPSS file *towel.sav* located on the FTP server in the Stat337 directory.

15.6   Displaying the Data

SPSS produces the following line chart of mean amount of water absorbed versus time by towel brand:
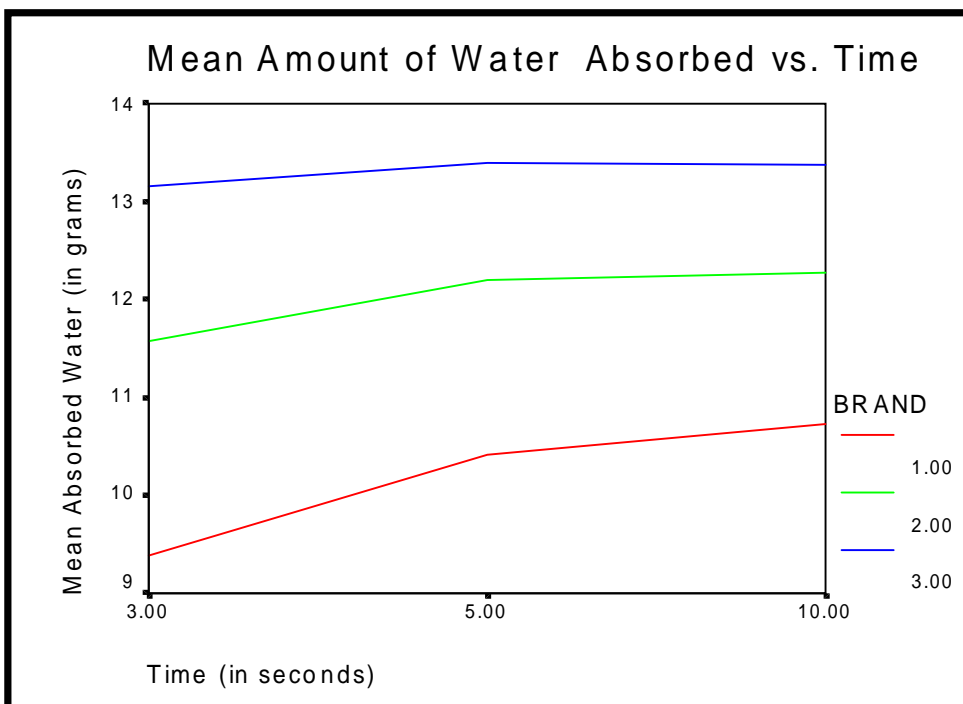
The lines in the graph are obtained by connecting the factor-level means for the three time levels. The plot indicates that the mean amount of water absorbed increases with the time level for all the three types of towels.

The mean amount of water absorbed is largest for the brand 3, smaller for the brand 2, and smallest for the brand 1 at all three time levels. The lines for the brands 2 and 3 are almost flat which indicates relatively small effects of time for the two brands. In other words, saturation has been achieved for the two brands almost immediately after putting the sheet from either of the two brands into water. The water is absorbed by the brand 1 slower, time plays more significant role in this case. The graph indicates very weak interaction between the means for the different brands when taken across the time levels.

15.7   Two-Way ANOVA

 The absorbency of paper towels experiment is an example of a factorial experiment. A factorial experiment consists of several factors (brand, immersion time) which are set at different levels, and a response variable (weight of water absorbed). The purpose of the experiment is to assess the impact of different combinations of the levels of brand and time factors on the weight of water absorbed by paper towel.

The General Factorial Procedure available in SPSS 8.0 provides regression analysis and analysis of variance for one dependent variable by one or more factors or variables. The SPSS data file used for this study is available in the SPSS file *towel.sav* located on the FTP server in the Stat337 directory. In the data file, variables include brand, time and weight of water absorbed. The two-predictor variables in this study, brand type and time level, are categorical, which means they should be entered as factors in the GLM General Factorial procedure.

To produce the output for your data, select *SPSS Instructions* in the problem menu now. Here, we will display and analyze the output for our data.

Analysis of variance allows us to test the null hypothesis that brand type and time have no impact on absorption. There are four sources of variation in the experiment: the main effects of *Brand* and *Time*, the interaction effect, and the error variation. Corresponding to these four sources, there are three null hypotheses that may be tested:

1. H_0: No main effect of *Brand*
2. H_0: No main effect of *Time*
3. H_0: No interaction effect between *Brand* and *Time*

The GLM General Factorial procedure in SPSS produces the following output for the experiment:

Tests of Between-Subjects Effects

Dependent Variable: WATER

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	81.303ª	8	10.163	58.632	.000
Intercept	6303.617	1	6303.617	36367.021	.000
BRAND	74.712	2	37.356	215.517	.000
TIME	4.915	2	2.458	14.178	.000
BRAND * TIME	1.676	4	.419	2.417	.067
Error	6.240	36	.173		
Total	6391.160	45			
Corrected Total	87.543	44			

a. R Squared = .929 (Adjusted R Squared = .913)

The table contains rows for the components of the model that contribute to the variation in the dependent variable. The row labeled *Corrected Model* contains values that can be attributed to the regression model, aside from the intercept. The sources of variation are identified as *Brand*, *Time*, *Brand*Time* (interaction), and *Error*. *Error* displays the component attributable to the residuals, or the unexplained variation. *Total* shows the sum of squares of all values of the dependent variable. *Corrected Total* (sum of squared deviations from the mean) is the sum of the component due to the model and the component due to the error.

According to the output the model sum of squares is 81.303 and the error sum of squares is 6.240. The total sum of squares (corrected total) is 87.543. Notice a very small contribution of error in the total sum of squares. The p-value of the F-test for the model is reported as 0.000 indicating convincing evidence of an effect of at least one of the factors on absorbency.

The sum of squares for the brand factor is estimated to be only 74.712, an extremely large value compared to the total sum of squares. The value of the F-statistic equal to 215.517 and p-value of the F-test reported as 0.000 indicate very strong evidence of effect of brand on absorbency. Indeed, in all graphical displays and numerical summaries we found strong evidence of brand effect on absorbency.

The sum of squares due to time is also 4.915, a very small contribution in the total sum of squares of 87.543. The value of the F-statistic is 14.178 with the corresponding reported p-value of 0.000. *Time* main effects are also statistically significant, although they are not that strong as the main effects due to brand factor.

The p-value of the interaction term *Brand*Time* is equal to 0.067, indicating weak evidence of an interaction between the two factors.

To further explore the interaction effects, we examine the table of estimated marginal means and the profile plot of the same values displayed below.

Report

WATER

BRAND	TIME	Mean	N	Std. Deviation
1.00	3.00	9.3800	5	.6419
	5.00	10.4200	5	.6058
	10.00	10.7200	5	.1924
	Total	10.1733	15	.7658
2.00	3.00	11.5800	5	.4712
	5.00	12.2000	5	.4528
	10.00	12.2800	5	.1304
	Total	12.0200	15	.4814
3.00	3.00	13.1600	5	.4037
	5.00	13.4000	5	.3162
	10.00	13.3800	5	.1924
	Total	13.3133	15	.3137
Total	3.00	11.3733	15	1.6739
	5.00	12.0067	15	1.3408
	10.00	12.1267	15	1.1411
	Total	11.8356	45	1.4105

The table shows the means of amount of water absorbed for each combination of levels of the two factors. The mean amount of water absorbed increases when taken across time levels. There is almost no change in the mean when time moves from the 5 second level to the 10 second level.

The means for each brand at the time level of 5 seconds differ very little from the overall mean for each brand, which is why the time main effects seem to be statistically significant. On the other, the differences for the time levels of 5 and 10 seconds are very small for each brand.

The overall means among the three brands, 10.1733, 12.0200, and 13.3133 differ significantly, which indicates very strong brand main effects.

As you can see, the standard deviation for each brand decreases when taken across the three time levels.

The table shows that the highest mean amount of water absorbed is achieved for the brand 3. The lowest mean amount of water absorbed is achieved for the brand 1.

The results in the ANOVA table in the previous section clearly indicate a difference among the means for the brand and the time factors, but they do not identify just which mean differs from another. The results of the Tukey method for each factor are presented below.

Multiple Comparisons

Dependent Variable: WATER

Tukey HSD

(I) BRAND	(J) BRAND	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1.00	2.00	-1.8467*	.152	.000	-2.2183	-1.4751
	3.00	-3.1400*	.152	.000	-3.5116	-2.7684
2.00	1.00	1.8467*	.152	.000	1.4751	2.2183
	3.00	-1.2933*	.152	.000	-1.6649	-.9217
3.00	1.00	3.1400*	.152	.000	2.7684	3.5116
	2.00	1.2933*	.152	.000	.9217	1.6649

Based on observed means.

  *. The mean difference is significant at the .05 level.

As you can see, there are significant differences among the three brands. The p-values for all the comparisons between the brands are reported as zero.

The results of the Tukey test for the time factor are displayed below.

Multiple Comparisons

Dependent Variable: WATER

Tukey HSD

(I) TIME	(J) TIME	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
3.00	5.00	-.6333*	.152	.001	-1.0049	-.2617
	10.00	-.7533*	.152	.000	-1.1249	-.3817
5.00	3.00	.6333*	.152	.001	.2617	1.0049
	10.00	-.1200	.152	.712	-.4916	.2516
10.00	3.00	.7533*	.152	.000	.3817	1.1249
	5.00	.1200	.152	.712	-.2516	.4916

Based on observed means.

  *. The mean difference is significant at the .05 level.

The above results confirm the conclusions we have reached before. The difference between the time level of 3 seconds and 5 seconds is significant with the p-value of .001. The same holds for the time level of 3 seconds and 10 seconds. However, the difference between the time levels of 5 and 10 seconds is found to be non-significant. Indeed, the graphical displays discussed in Section 7 indicated that the paper towels absorb water very fast. This is why there is a significant difference in the mean weight of water absorbed between time level of 3 seconds and time level of 5 seconds. Then a degree of saturation has been achieved, and there is very little change in the amount of water absorbed as time elapses.

15.8   Summary

The goal of the experiment is to compare the absorbency of three brands of paper towels. The factors are brand of paper towel and time it takes a sheet from the brand to absorb a specific amount of water. The response variable is the weight of water absorbed by the sheet. Observe that including time as a factor allows you to measure not only how much water a sheet holds (absorbency) but also how fast it absorbs water.

In this experiment, three paper towel brands and three time levels are used, so it is a 3x3 factorial experiment, meaning two factors at three levels each. There are nine combinations of the levels of the factors, and because there are five observations in each combination, there are five replications and overall 45 measurements. To avoid bias, the 45 absorbency measurements are taken according to a random allocation of combinations.

The experiment is an example of a factorial experiment. The GLM General Factorial Model in SPSS was used to analyze the data. The main effects of brand and time were found to be statistically significant. However, the comparison of the magnitudes of the F-statistic for the two models indicates that the main effects due to brand are much stronger than the effects due to time.

The analysis showed very weak interaction between brand and time. Indeed, the brands 2 and 3 absorbed almost all its water during the first few seconds after the sheets from these two brands were immersed in water. On the other hand, the relationship between the weight of water absorbed and time was much stronger for the brand 1. In this case water was absorbed slower compared to the other brands.

The profile plots and numerical summaries show that the highest mean absorption measured by the weight of water absorbed is achieved by the brand 3 at every time level. The analysis also showed that the brand absorbed water much faster than the other brands.