8. The Nonparametric Approach

The paired t-test presented in the previous section has an assumption of normality for the differences. The normal quantile plot discussed in Section 6 indicates that the assumption of normality might be slightly violated for these data. In this section two distribution-free alternatives to the paired t-test will be used.

8.1 The Wilcoxon Signed-Rank Test
8.2 The Sign Test

8.1 The Wilcoxon signed-rank test does not require normality of the differences, but it is assumed that the differences are independent. Moreover, this test assumes that the distribution of the differences is symmetric.

The Wilcoxon signed-rank test can be used with normal and nonnormal data and is often more powerful than the paired t-test when the population is not normal. It is worthy to remind you that the power of a test is the probability of rejecting a false null hypothesis. A good test has high power.

We define our hypotheses in the same way as before:

H_0: $\mu_D=0$ ($\mu_A-\mu_B=0$) versus H_a: $\mu_D>0$ ($\mu_A-\mu_B>0$).

SPSS produces the following output:

Wilcoxon Matched-Pairs Signed-Ranks Test

Mean Rank	Sum of Ranks	Cases
11.91	190.5	16 - Ranks (BRAND_B LT BRAND_A)
4.88	19.50	4 + Ranks (BRAND_B GT BRAND_A)
		0 0 Ties (BRAND_B EQ BRAND_A)

Number of observations 20

Z = -3.1925 2-Tailed P = .0014

The p-value of the two-tailed test is 0.0014. Thus the p-value of our one-sided test is 0.0014/2=0.0007. That small p-value indicates strong evidence against the null hypothesis in favour of the alternative $\mu_A-\mu_B>0$. However, observe that the assumption of symmetry for the distribution of differences is questionable. In our sample, 16 out of 20 differences are positive and only 4 are negative.

8.2 The Sign-Test counts the number X of pairs where one group's measurement exceeds the other's. In our experiment, the tread depth of brand A tires was larger than the tread depth of brand B tires for X=16 out of the n=20 pairs. If there is no difference between the tread depths of brand A and brand B tires (the null hypothesis is true), the distribution of X is binomial with n=20 trials and p=0.5, where p is the probability of success in a single trial. Thus the probability of the outcome as large as the observed or larger is

$P(X \geq 16|n=20, p=0.5)=1-P(X \leq 15| n=20, p=0.5)=0.0059.$

This value is the p-value for the one-sided alternative.

The identical result can be obtained using SPSS:

```
Sign Test

BRAND_A with BRAND_B

 Cases

  16  - Diffs (BRAND_B LT BRAND_A)
   4  + Diffs (BRAND_B GT BRAND_A)
   0    Ties
  --                          (Binomial)
  20    Total                Exact 2-Tailed P =  .0118
```

SPSS provides the p-value for the two-sided alternative. The p-value for the one-sided alternative can be obtained by dividing the value by two: 0.0118/2=0.0059.