CLOUD SEEDING EXPERIMENT

8. Tests of Significance and Confidence Intervals

We will apply the t-tools on the log scale because the log-transformed rainfalls have distributions that appear satisfactory for using the tools. The t-test and confidence interval will be constructed in the usual way but on the transformed data. Click on one of the three bolded links below to access the appropriate topic.

8.1 SPSS output.
8.2 Inferences on the log scale.
8.3 Inferences on the original scale.

8.1 SPSS produces the following output:

t-tests for Independent Samples

Variable	Number of Cases	Mean	SD	SE of Mean
RAINFALL				
Unseeded	26	3.9904	1.642	0.322
Seeded	26	5.1342	1.600	0.314

Mean Difference = -1.1438

Levene's Test for Equality of Variances: F= .058 P= .811

t-test for Equality of Means

Variances	t-value	df	2-Tail Sig	SE of Diff	95% CI for Diff
Equal	-2.54	50	.014	.450	(-2.047, -.241)
Unequal	-2.54	49.97	.014	.450	(-2.047, -.241)

The instructions how to obtain the output are provided in Section 12 (click here to access them).

The output starts with statistics of the two groups, followed by the value of the difference between means. The Levene test for equality of variances is also included. Provided the F value is not significant ($P>0.05$), the variances can be assumed to be equal and the *Equal Variances* line of values for the t-test can be used. If $P<0.05$, then the equality of variances assumption has been violated and the t-test based on unequal variances should be used.

In our case, the high P-value of 0.811 in the Levene's Test for equality of variances strongly indicates that the data are consistent with the equality of variances assumption.

8.2 Now we will interpret the outcome of the test and the confidence interval.

We test the null hypothesis that there is no seeding effect for the log-transformed observations (treatment effect is zero). In other words, we test the claim that there is no effect of cloud seeding on log rainfall.

A suitable tool to test the hypothesis of no treatment effect is the two-sample t-statistic. The *Independent-Samples T Test* procedure available in SPSS assesses the significance of the effect of cloud seeding on log rainfall. Denote by δ the additive effect of cloud seeding on the logarithm of rainfall. Thus the null and alternative hypotheses are

H_0 : $\delta = 0$ (no additive effect of cloud seeding on the log rainfall),

H_a : $\delta > 0$ (evidence of additive effect of cloud seeding on the log rainfall).

The value $\hat{\delta}$ defined as the difference between the average rainfall for unseeded clouds and the average rainfall for seeded clouds is an estimate $\hat{\delta}$ of δ. The t statistic under the null hypothesis has the form

$$t = \frac{\hat{\delta} - \delta}{SE(\hat{\delta})} = \frac{\hat{\delta} - 0}{SE(\hat{\delta})}.$$

The P-value of the two-sided t-test with the assumption of equal variances is obtained by SPSS as 0.014. Hence, one-sided p-value is $0.014/2 = 0.007$. That means that there is convincing evidence to reject the null hypothesis of no effect of cloud seeding on log rainfall. The data support the claim that seeding causes the increase in rainfall.

The P-value in this case is the probability that randomization alone leads to a test statistic as extreme or more extreme than the one observed, if in fact cloud seeding had no effect on rainfall. The smaller the p-value, the more unlikely it is that the random allocation of clouds to the two treatment groups (seeded, unseeded) is responsible for the discrepancy between rainfalls, and the greater the evidence that the null hypothesis is incorrect.

We have applied the t-tools on the log scale because the log-transformed rainfalls have distributions that appear satisfactory for using the tools. Now we will transform our estimates back to the original scale.

According to the above output, the average seeded log rainfall minus the average unseeded log rainfall is 1.1438. The logarithm transformation enables us to obtain an estimate of the multiplicative effect of cloud seeding on rainfall.

In general, if Y_1, Y_2 are the responses to treatments 1 and 2, and $\bar{Z}_1, \bar{Z}_2$ are the log transformed averages for the two treatments, then

$$\exp(\bar{Z}_2 - \bar{Z}_1) \quad \text{estimates} \quad \frac{response(treatment2)}{response(treatment1)}.$$

In other words, the value of $\exp(\bar{Z}_2 - \bar{Z}_1)$ estimates how many times the response to treatment 2 is as large as the response to treatment 1. For details see your textbook, page 67-68.

In our case, the difference between the average rainfalls for seeded and unseeded clouds is $\bar{Z}_2 - \bar{Z}_1$ = 1.1438, and therefore exp(1.1438)=3.1384 is an estimate of the ratio of the responses. Thus the volume of rainfall produced by a seeded cloud is estimated to be 3.14 times as large as the volume that would have been produced in the absence of seeding.

The 95% confidence interval for the difference between the logarithms for seeded and unseeded rainfalls is 0.241 to 2.047. Thus, the 95% confidence interval for the multiplicative treatment effect on the original scale is exp(0.241)=1.2720 to exp(2.047)=7.7425. In other words, the treatment effect is estimated to be between 1.27 and 7.74 times.

8.3 Now we discuss the outcome of the test on the original scale of measurement. The null hypothesis that there is no seeding effect for the log-transformed observations (additive treatment effect is zero) is equivalent to the hypothesis that there is no treatment effect on the original scale of measurement (multiplicative treatment effect is one). Obviously the p-value of the test with the log-transformed data is not affected by the operation.