

FAILURE TIMES OF BEARINGS

7. Multiple Comparisons

The experiment is an example of an unplanned experiment. Indeed, we didn't have any particular pairs to compare in mind before conducting the experiment. There is no particular structure to these groups, and hence the problem asks for a search through all the paired differences. This can be achieved by using multiple comparisons.

Multiple comparison procedures have been developed as ways of constructing individual confidence intervals so that the simultaneous confidence level is controlled (at 95%, for example). The 95% simultaneous confidence level means that we can be 95% confident that all the intervals simultaneously contain the differences. That is, in 95% of all experiments, every confidence interval would include the true value of $\mu_i - \mu_j$, and only 5% of the time would at least one interval fail to cover the true value.

SPSS has several multiple comparison procedures that should be run after the experiment has been conducted. The most important are Tukey's HSD method, Bonferroni method, the LSD (least significant difference) Fisher's method, and Duncan's method.

We will employ the Tukey's HSD method to detect significant differences between the group means. SPSS output for our data is displayed below:

Variable TIME	
By Variable CODE	
Multiple Range Tests: Tukey-HSD test with significance level .050	
The difference between two means is significant if $\text{MEAN}(J) - \text{MEAN}(I) \geq 3.1609 * \text{RANGE} * \text{SQRT}(1/N(I) + 1/N(J))$ with the following value(s) for RANGE: 4.02	
(*) Indicates significant differences which are shown in the lower triangle	
	GROUP 2 3 4 1 5
Mean	CODE
6.0500	2
8.6360	3
9.7980	4
10.6930	1
14.7060	5
	* *

Thus the experiment has shown that the material in the group 5 (Powder processed EX00007) is significantly superior to both the material in the group 2 (CEVM M-50) and the material in the group 3 (Powder processed AISI-T-15). On the other hand, the material in the group 5 cannot be claimed to be superior to any material from the other three groups. Statements of a similar type can be made for each material.

SPSS also provides the homogeneous subsets (means not significantly different) for the case study:

Homogeneous Subsets (highest and lowest means are not significantly different)				
Subset 1				
Group	Grp 2	Grp 3	Grp 4	Grp 1
Mean	6.0500	8.6360	9.7980	10.6930
Subset 2				
Group	Grp 4	Grp 1	Grp 5	
Mean	9.7980	10.6930	14.7060	

We will use the case study to demonstrate the relationship between the multiple comparisons results and the confidence intervals generated by the same multiple comparisons method. More precisely, we will show that if one of the 95% confidence intervals includes the value 0, then that pair of means will not be declared significantly different, and vice versa.

The above output provides the value of the 95th percentile from the Studentized range distribution as RANGE= 4.018. Thus 95% confidence intervals for differences between means all have halfwidth equal to

$$\text{HALFWIDTH} = 3.1609 * 4.018 * \text{SQRT}\left(\frac{1}{10} + \frac{1}{10}\right) = 5.679835 \sim 5.68.$$

Therefore, 95% confidence intervals for differences between means have the form

$$(\bar{x}_i - \bar{x}_j) \pm 5.68.$$

The table of the differences between all group means (rounded to two digits after the point) has the form:

GROUPS	AVERAGE	DIFFERENCES FROM GROUP			
		2	3	4	1
5	14.71	8.66*	6.07*	4.91	4.01
1	10.69	4.64	2.06	0.90	
4	9.80	3.75	1.16		
3	8.64	2.59			
2	6.05				

The two differences where the corresponding confidence interval excludes zero appear with the asterisks. These two differences have been detected by the test discussed before.