Department of Mathematical and Statistical Sciences

Stat 366 Asg 1

Due on Sept 22 2006

1. Of the travelers arriving at a small airport, 60% fly on major airlines, 30% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50% are traveling for business reasons, whereas 60% of those arriving on private planes and 90% of those arriving on other commercially owned planes are travelling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person

   (a) is travelling on business?

   (b) is travelling for business on a privately owned plane?

   (c) arrived on a privately owned plane, given that the person is travelling for business reasons?

2. A salesperson can contact either one or two customers per day with probability 1/3 and 2/3, respectively. Each contact will result in either no sale or a \$100 sale, with the probabilities 0.8 and 0.2, respectively. Find the probability distribution for daily sales. Find the mean and variance of the daily sales.

3. The length of time Y to complete a key operation in the construction of houses has an exponential distribution given by $f(y) = \frac{1}{10}\, e^{-y/10}$, $y > 0$. The formula $C = 40Y + 100$ relates the cost of completing this operation to the time to completion Y. Find the mean and variance of C. Would you expect C to exceed 2000 hours very often? Explain.

4. Suppose that $Y_1, Y_2, ..., Y_{40}$ denote a random sample of measurements on the proportion of impurities in iron ore samples. Let each variable Y_i have a probability density function given by

   $$f(y) = 12y^2(1-y), \;\; 0 \leq y \leq 1.$$

   The ore is to be rejected by the potential buyer if the sample mean $\overline{Y}$ exceeds 0.7.

   (i) Find the probability of rejecting the ore by the buyer.

   (ii) If 10 different samples of size 40 are taken from iron ore production, what is the probability that at least 3 of them will have to be rejected?

5. Let $Y_1, Y_2, ..., Y_n$ denote a random sample from a Poisson distribution with mean λ.

   (i) Suggest an unbiased estimator for λ.

   (ii) Define $C = 3Y + Y^2$. Show that $E(C) = 4\lambda + \lambda^2$.

   (iii) Find a function of $Y_1, Y_2, ..., Y_n$ that is an unbiased estimator of $E(C)$.

6. Let $Y_1, Y_2, ..., Y_n$ denote a random sample of size n from a population with whose density is given by

   $$f(y) = \frac{5y4}{\theta^5} \;,\;\; 0 \leq y \leq \theta,$$

   where $\theta > 0$ is unknown. Consider two estimators $\widehat{\theta}_1 = \frac{6}{5}\overline{Y}$ and $\widehat{\theta}_2 = \frac{5n+1}{5n} Y_{(n)}$, where $\overline{Y} = \frac{1}{n}\sum Y_i$ and $Y_{(n)} = \max\,(Y_1, Y_2, ..., Y_n)$. Find the efficiency of $\widehat{\theta}_1$ relative to $\widehat{\theta}_2$. Which one do you recommend?

7. Let $Y_1, Y_2, ..., Y_n$ denote a random sample of size n from a population with whose density is given by

   $$f(y) = \frac{3\theta^3}{y^4} \;,\; y \geq \theta,$$

   where $\theta > 0$ is unknown. Consider the estimator $\widehat{\theta} = \min\,(Y_1, Y_2, ..., Y_n)$. Find the mean squared error (MSE) of $\widehat{\theta}$.

8. Let $Y_1, Y_2, ..., Y_n$ denote a random sample of size n from a normal distribution with mean θ_1 and variance θ_2. Find the maximum likelihood estimators of θ_1, θ_2 and $\frac{\theta_1}{\theta_2}$.